Visualization of Multivariable Calculus

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Abstract: The maxima and minima of functions of single, double and multi variable are comprehensible to some extent with classical and the conventional approach. The nature of extremum can be viewed and visualize manually for the simple functions of single variable with ease through graphical representation where as it is quite difficult to view nature of maxima and minima for complex curves and surfaces. ICT tools are playing a vital role in Teaching Learning Process, with the advent of mathematical softwares, it is easy to visualize and represent simple to complex functions effectively. This paper presents method and the procedure for finding extrema for the functions of single and multi-variables supported by Geogebra in order to visualize stationary points or critical points, saddle points and points of extremum. Examples are solved and represented graphically. The main aim of the paper is to provide firsthand information about the usage of Geogebra software in visualizing the maxima and minima of function of single and multi-variables functions as well as to make classroom attractive with latest digital tools. It useful for teacher and students in grasping full comprehensibility about the subtle nuances and variations in cuves and surfaces with respect to the maxima and minima.

Keywords: Geogebra, computer-assisted instruction, maxima and minima.

Introduction

The process of representation comprises the utilization of different models for the sake of memorizing, organizing and exchange of mathematical concepts. The aim and objective is to grasp complete comprehensibility of mathematics ideas and concepts so that conclusion can be drawn for the problem.

Many investigators contributed to the Representation, illustration and graphical view of mathematical concepts like functions etc. It is the accepted fact that teaching learning process will be enhanced with the help of suitable representation of concepts. The advent of computer algebra systems gives new dimension to the teaching and learning of Calculus which is quite different from classical and monotonic symbolic techniques and methods. This enables the learner to develop high level cognitive skill with more emphasize on conceptual learning and problem solving ([1, 2, 3, 4,6, 7, 11]).

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Mathematics play vital role in different fields of technology. The subtle nuances of mathematical concepts can be better understood and viewed through graphical representation in different ways. It is very effective from lower level to higher level whether it is connected to teaching learning process or in the field of research. The literature review endorses that use of technology in teaching and learning beneficial in all aspects further it enhances mental ability and capability of the student particularly in learning mathematical concepts and ideas. ([1, 3, 4, 8, 10, 11]). Calculus always be the topic of extensive concern for researchers over long period of time. It is noticed from literature review that students faced cognitive complications and difficulties in grasping the mathematical concepts pertaining to Differentiation, Multiple Integration, curvature, envelop, Maxima and Minima etc. which are considered to play crucial role engineering and technology. ([1, 2, 3]).

Visualization of Maxima and Minima of Function

In the field of calculus, where maxima and minima of functions is one of the significant concepts, definition of maxima and minima should focus on two concepts: Graphically, the graph of *the function* which is required to find extrema subject to some condition or without constraints and the second, the stationary or the critical points of the functions.

Formally a function is said to have a relative maximum at some point if it possess the greatest value in the neighborhood of that point, similarly it attains relative minimum if it has lowest value in the neighborhood of that point. Generally the point of maxima or minima is called extremum. Algebraically for the function of single variable y=f(x) differentiating the function and equating to zero yields set of critical or the stationary values that is considered as necessary condition for maxima and minima. Differentiating the same function second time and checking the positivity or the negativity of the resultant value decides minimum or maximum point.

That is $f^{11}(x) > 0$ gives point of minimum. Where as $f^{11}(x) < 0$ gives point of maximum.

For example:

$f(x) = 0.93x^3 - 2x^2 - 3x + 3$

f has maximum at x=-0.54 and maximum value is 3.84 and

f has minimum at x=1.98 and minimum value is -3.56

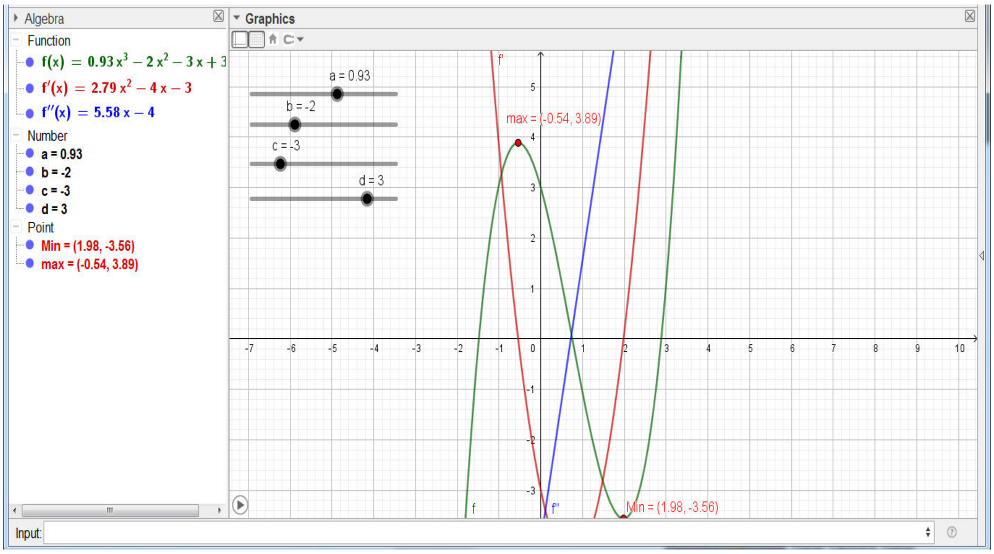

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Fig 1: Finding Max/Min of $f(x) = 0.93x^3-2x^2-3x+3$

The same problem can be solved by using geogebra tool in an effective manner and graphically representation for maximum and minimum can be visualized from different views. Instead of defining the specific function

$f(x) = 0.93x^3-2x^2-3x+3$

The family of curves can be defined as

$f(x) = ax^3+bx^2+ cx+d$

By assigning different values to the parameters a,b,c, and d the maxima and minima of family of curves can be illustrated

For the above defined function in the example

a = 0.9 ,b = -2 , c = -3 and d = 3

The above figure depicts maxima and minima of the function $f(x)=0.93x^3-2x^2-3x+3$ graphically.

The following is the figure representing maxima and minima of the function $f(x)=ax^3+bx^2+ cx+d$ where a = 1.3, b = -1.4, c = -2.2, d = 3

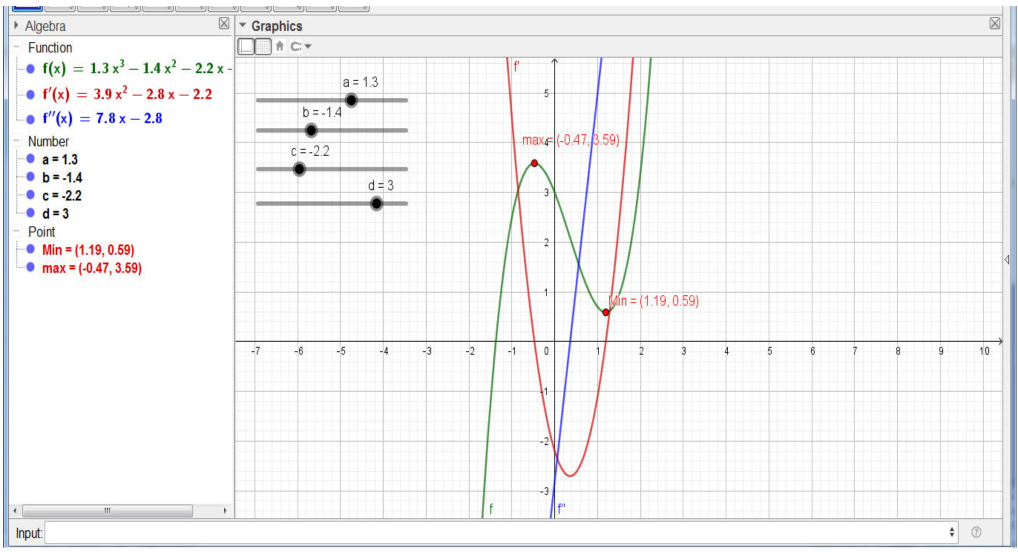

Fig 1: Finding Max/Min of $f(x) = 1.3x^3-1.4x^2-2.2x+3$

In case of function of two variables, geometrically z=f(x,y) represents a surface in three dimensional space. The maximum point is the point lies on the surface (hilltop) from which the surface comedown in all direction. The minimum is the bottom or the point of depression from which the surface goes up in all direction. Saddle point or minimax is not the point of extrema. It is the point at which the function is neither attains maximum nor minimum. Basically at the saddle point the function f seems to be maximum in one direction whereas minimum in other direction.

Analytically procedure for finding maxima and minima of the function of two variables is as follows
:
Solving the equations f_x =0 and f_y =0 gives stationary or the critical points P of the function.
then calculating r = f_{xx} , s = f_{xy} t = f_{yy} at point P.
The P is maximum if $rt-s^2 > 0$ and $r < 0$ then f attains maximum at P

The P is minimum if $rt-s^2 > 0$ and $r > 0$ then f attains minimum at P
The P is a saddle point if $rt-s^2<0$ then f attains neither maximum nor minimum
The failure case arises if $rt-s^2=0$ then further investigation is needed.

The concept can be extended to multivariable calculus and visualized from different views. The behavior of simple to complex functions can be visualized graphically and it will pave the way for the learner to grasp complete comprehensibility of the concepts through software tools. Generally as far as graphs of functions of two and three variable are concerned it is very cumbersome task to get complete understanding analytically but it can be viewed clearly with the help of software and computer assisted learning tools.

CONCLUSION:

The computer algebra system have the quality to facilitate a dynamic approach teaching and learning process and allow the students to participate in innovating and accumulate their knowledge,therefore building up geometrical and conceptual comprehensibility by providing deeper insight into the learning process with the advent of mathematical tools, abstract and complicated concepts mathematical equations can be explained with visualization in an effect manner.

The main objective of the paper was to focused on significance and importance of mathematical software in teaching and learning process. Geogebra software was used to solve some problems pertaining to maxima and minima of functions of single and it extension to two variables analytically as well as graphically. Use of mathematical tools in learning mathematical concepts not only boost the understanding of learners but also save time, avoid tedious and monotonous calculations procedures and process. In case of maxima and minima of functions of multi variables problems concept of one has to equipped with differentiation, solving equations, curve tracing, properties of curves etc local maxima and minima of functions, global or absolute maxima or minima different functions like polynomial, exponential, logarithmic, geometric or composite functions hyperbolic and special function is cumbersome task and time consuming with limited understanding manually or analytically .The problems which may be complex or simple in nature can be dealt easily with the help of mathematical tools and much more emphasize can be focused on graphical representation, nature of curves and surfaces. Local maxima and Minima of the functions can easily be visualized by simple assisting difference interval, or regions.

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