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COMBINED EFFECTS OF HEAT AND MASS TRANSFER ON MIXED CONVECTION FLOW OF AN INCOMPRESSIBLE VISCOUS FLUID WITH NON- COAXIAL ROTATION

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ABSTRACT: Heat and mass transfer in unsteady non-coaxial rotating flow of viscous fluid over an infinite vertical disk is derived and obtain the exact solution by Laplace transform technique. The motion in the fluid is induced due to two sources. Firstly, due to the buoyancy force which is caused because of non-coaxial rotation of a disk such that the disk executes cosine or since oscillation in its plane and the fluid is at infinity.The problem is modeled in terms of coupled partial differential equations with some physical boundary and initial conditions. The dimensionless form of the problem is solved via Laplace transform method for exact solutions Expressions for velocity field, temperature and concentration distributions are obtained, satisfying all the initial and boundary conditions. Skin friction, Nusselt number and Sherwood number are also evaluated. The physical significance of the mathematical results is shown in various plots and is discussed for several embedded parameters. It is found that magnitude of primary velocity is less than secondary velocity. In limiting sense, the present solutions are found identical with published results.

KEYWORDS: Viscous fluid, Non- coaxial rotation, Primary Velocity, Secondary Velocity, Heat and Mass transfer, exact solution

1. Introduction

The principal interest of this work is to study the convective transport of momentum, heat and mass.Convective transport is of three types namely forced, free and mixed. Forced convection occurs when the flow is caused either by external force or by imposing non-homogeneous boundary condition on velocity. Opposite to the forced convection, in natural or free convection, the transport phenomenon occurs due to buoyancy force that arises from density differences caused by temperature and concentration variations in the fluid. However, a situation where the free and forced convection mechanisms simultaneously and significantly contribute to the above transport phenomena is called mixed or combined convection.The combined convection phenomenon occurs in many technical and industrial problems such as electronic devices cooled by fans,nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, and solar collector[1,2].Over the time various publications on mixed convection with different boundary conditions and situations appeared For example, some of the most recent and interesting studies we discuss here.Hussanan et al.[3] investigated thermal diffusion,

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chemical reaction, heat absorption and Newtonian heating effects on mixed convection flow of viscous fluid with combined heat and mass transfer.Srinivasacharya and Reddy [4] studied chemical reaction and radiation effects on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium.Bhukta et al.[5] analyzed dissipation effect on MHD mixed convection flow over a stretching sheet through porous medium with non-uniform heat source/sink. Raju et al.[6] examined heat and mass transfer in MHD mixed convection flow on a moving inclined porous plate. Hayat et L. [7] studied three dimensional mixed convection flow of viscoelastic fluid with thermal radiation and convective heat transfer flow of the fluid with thermal radiation and convection heat transfer flow of the fluid with thermal radiation and convective conditions. Ellahi et al. [8] focused on mixed convection heat transfer flow of the fluid over wedge with a porous medium. Besides that, an analysis on the mixed convection flow under a constant heat flux through a square cavity with a wavy wall has been performed by Mamourian et al. [9].Babulal and Talukdar [10] investigated combined effects of Joule heating and chemical reaction on unsteady magneto hydrodynamic mixed convection of a viscous dissipating fluid over a vertical plate in porous media with thermal radiation. Similarly, Khan et al. [11] discussed the effects of heat and mass transfer on magneto hydrodynamics (MHD) flow in a porous channel. Samiluhaq et al [12] also investigated the phenomenon of heat and mass transfer with MHD flow past a vertical plate with ramped wall temperature. Then, Ali et al.[13] analyzed the combined processes of heat and mass transfer by considering the chemical reaction in the fluid flow. In addition, Nadeem et al.[14] examined the heat and mass transfer analysis of the fluid flow through eccentric cylinders and followed by Nadeem et al.[15],where a problem on heat and mass transfer over a vertical rectangular duct is investigated. The influence of tapered stenosed artery of permeable wall on combined heat and mass transfer in blood flow has been investigated by Ellahi et al. [16].The flow between two Non- parallel plane wllas with the effect of heat and mass transfer has been presented by Adan et al.[17].Other than that, Khan et al.[18] discussed heat transfer in the fluid flows between two parallel plates. This is the opposite physical study with [17]. Next, two related problems of heat and mass transfer have been solved by Khan et al.[19,20] but cent rated on permeable stretching surface saturated by porous medium with a convective boundary condition[19] and flow over a moving wedge with the effect of MHD[20].Besides that, Khan et al [21] studied the effect of heat transfer in rotating channel with lower stretching permeable wall.On the other hand, Ismail et al.[22,23] considered the rotating fluid in heat and mass transfer with the inclined plate.It was found that, as inclination angle increased, the fluid flow in primary and secondary flow was decreased. Islam et al [24], Muthucumaraswamy et al [25, 26] and Mohyud -Din et al.[27] also investigated the reaction of heat and mass transfer in a rotating fluid.

However, In the context of above background, the transport phenomenon of momentum, heat and mass is studied either in rotating or in non-rotating frame.In this work, we assume that this transport is in frame with non- coaxial rotation. This idea of non- coaxial rotation with heat and mass transfer over an oscillating disk is not investigated yet. However, similar studies of non- coaxial rotation, for only momentum transfer, are available in the literature. Among them, Erdogan [28] obtained an exact solution for non- coaxial rotation of viscous fluid through a porous disk Asghar et al [30] extended Hayat et al. [29] problem for accelerated porous disk. In addition Curia et al. [31] introduced a new knowledge of non-coaxial rotation by taking the hall current effect into hydro magnetic flow over an infinite porous disk. After that, an electrically conducting viscous fluid between two parallel eccentric rotating disks has been studied by Maji et al. [32].Curia et al. [33] and Ahmad et al

[34] introduced velocity slip in non-coaxial rotation. Recently, Das et al. [35,36] studied the effects of hall currents and slip condition on non-coaxial rotation viscous fluid through an infinite porous disk. Furthermore, Das et al. [37] concentrated on the problem of eccentric concentric rotation of a disk due to non- coaxial rotating also has been investigated by Lakshmi et al [38] and Ersoy et al.[39-41]. An interesting problem of non- coaxial rotation with heat transfer has been solved by Mohammad et al. [42] where they studied the effect of free convection on fluid motion.

Based on the above discussion, the present work aims to study the combined effects of heat and mass transfer on mixed convection flow of an incompressible viscous fluid over an oscillating an infinite vertical disk with non- coaxial rotation and fluid at infinity.The problem is first modeled and then solved for the exact solution using Laplace transform technique. Results are plotted and discussed for differ parameters of interest.

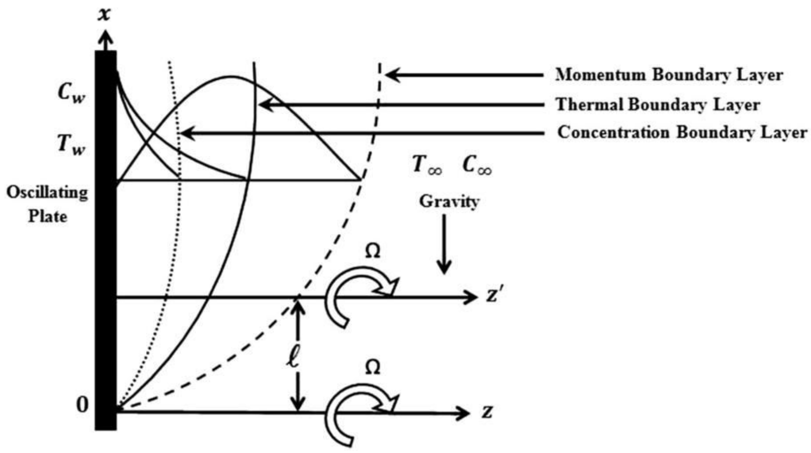

Figure 1

2. Mathematical formulation of the problem

Let us consider a Cartesian coordinate system with z-axis normal to a rigid disk. The x-axis is taken in upward direction along the disk and z-axis is taken normal to the plane of the disk. The axes of rotation for both the disk and fluid are assumed to be in plane x =0. Initially, at t= 0, the disk and fluid at infinity are rotating about z' axis with the same angular velocity Ω with temperature T_∞ and concentrationC_∞. After time t>0, the disk suddenly starts to oscillate and rotates about z-axis with uniform angular velocityΩ while the fluid at infinity continues to rotate about z'-axis with the same angular velocity as that of the disk. The temperature of the disk and concentration raise to $T_\infty$and C_∞ respectively. The distance between the two axes of rotation is equal to l. The physical sketch of the problem is shown in Fig.1. Under the above assumptions, we seek solutions for velocity field, temperature and concentration distributions of the forms

$$V = (u(z,t), v(z,t), 0), T = T(z,t) and C = C(z,t), \quad (1.1)$$

$$u(z,t) = -\Omega y + f(z,t), v(z,t) = -\Omega x + g(z,t) \quad (1.2)$$

Under the above assumptions and by using the usual Boussinesq approximations the equations governing the flow [11, 12, 23, 25, 42]

$$\frac{1}{\rho}\frac{\partial p}{\partial x} - \Omega^2 = v\frac{\partial^2 f}{\partial z^2} - \frac{\partial f}{\partial t} + \Omega g + g_x\beta_T(T - T_\infty) + g_x\beta_c(C - C_\infty) \quad (1.3)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} - \Omega^2 y = v\frac{\partial^2 f}{\partial z^2} - \frac{\partial g}{\partial t} - \Omega f \quad (1.4)$$

$$\rho c_p\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} \quad (1.5)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2} \quad (1.6)$$

With boundary conditions corresponding to [11, 12,23, 25,42]

$$u(0,t) = -\Omega y + UH(t)\cos(\omega t)\, or$$

$$u(0,t) = -\Omega y + Usin(\omega t); \forall t > 0$$

$$v(0,t) = \Omega x;\ \forall t > 0,$$

$$T(0,t) = T_w;\ \forall t > 0,$$

$$C(0,t) = C_w;\ \forall t > 0, \quad (1.7)$$

$$u(\infty,t) = -\Omega(y - l), v(\infty,t) = \Omega x, T(\infty,t) = T_\infty,$$

$$C(\infty,t) = C_\infty;\ \forall t > 0, \quad (1.8)$$

And initial conditions :

$$u(z,0) = -\Omega(y - l), v(z,0) = \Omega x, T(z,0) = T_\infty,$$

$$C(z,0) = C_\infty;\ \forall t > 0, \quad (1.9)$$

where ρ is density of fluid, p is the pressure, v is the kinematic viscosity β_T and β_C are the coefficient of thermal expansion for temperature and concentration, g_x is the gravitational acceleration in x-direction, $T = T(z,t)$ is the temperature, c_p is the specific heat, k is the thermal conductivity, $C = C(z,t)$ is the concentration, D is mass diffusivity, H(t) is Heaviside function, ω is a frequency of oscillation, U is the characteristic velocity in x and y-directions.

By taking $x^2 + y^2 = r^2, \hat{p} = p - \frac{\rho}{2}\Omega^2 r^2$ as the modified pressure that equation (1.1)-(1.2) take the following forms:

$$v\frac{\partial^2 f}{\partial z^2} - \frac{\partial f}{\partial t} + \Omega g + g_x\beta_T(T - T_\infty) + g_x\beta_c(C - C_\infty) = \frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} \quad (1.10)$$

$$v\frac{\partial^2 g}{\partial z^2} - \frac{\partial g}{\partial t} - \Omega f = \frac{1}{\rho}\frac{\partial \hat{p}}{\partial y} \quad (1.11)$$

Where $\frac{\partial \hat{p}}{\partial x}$ and $\frac{\partial \hat{p}}{\partial y}$ are the modified pressure gradients. Differentiating eqs (1.10) and (1.11) with respect to z and using $\frac{\partial \hat{p}}{\partial z} = 0$, we get:

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$$\frac{\partial}{\partial z}\left[v\frac{\partial^2 f}{\partial z^2}-\frac{\partial f}{\partial t}+\Omega g+g_x\beta_T(T-T_\infty)+g_x\beta_c(C-C_\infty)\right]=0$$

Integration with respect to z

$$v\frac{\partial^2 f}{\partial z^2}-\frac{\partial f}{\partial t}+\Omega g+g_x\beta_T(T-T_\infty)+g_x\beta_c(C-C_\infty)=c_1(t) \tag{1.12}$$

Differential equations (1.10) with respectto z

$$\frac{\partial}{\partial z}\left[v\frac{\partial^2 g}{\partial z^2}-\frac{\partial g}{\partial t}-\Omega f\right]=\frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial \hat{p}}{\partial y}\right]$$

Apply $\frac{\partial p}{\partial z}=0$

$$\frac{\partial}{\partial z}\left[v\frac{\partial^2 g}{\partial z^2}-\frac{\partial g}{\partial t}-\Omega f\right]=0$$

Integration (1.11) with respect to z

$$v\frac{\partial^2 g}{\partial z^2}-\frac{\partial g}{\partial t}-\Omega f=c_2(t) \tag{1.13}$$

Where $c_1(z,t)$ and $c_2(z,t)$ are constant values. Since the fluid at infinity has no shear stress, all the derivatives of f and g are zero. After using Equation (1.8), Equation (1.12) and (1.13) reduce to the following forms:

$$u(\infty,t)=-\Omega(y-l)\,,v(\infty,t)=\Omega x,T(\infty,t)=T_\infty,$$

$$C(\infty,t)=C_\infty;\ y\to 0\ andx\to 0$$

$$u(\infty,t)=\Omega l;\ \frac{\partial u}{\partial t}=0\ \ \frac{\partial^2 u}{\partial z^2}=0,v(\infty,t)=0;\ \frac{\partial v}{\partial t}=0\ \ \frac{\partial^2 f}{\partial z}=0$$

$$T(\infty,t)=T_\infty;\ \frac{\partial T}{\partial t}=0\ \ \frac{\partial g}{\partial z}=0,C(\infty,t)=C_\infty;\ \frac{\partial C}{\partial t}=0\ \ \frac{\partial g}{\partial z}=0$$

When applying condition

$$c_1(t)=0$$

$$u(0)-0-\Omega(\Omega l)=c_2(t)$$

$c_2(t)=-\Omega^2 l$ Sub in (1.13) equation (1.12) and (1.13) becomes

$$v\frac{\partial^2 f}{\partial z^2}-\frac{\partial f}{\partial t}+\Omega g=-g_x\beta_T(T-T_\infty)-g_x\beta_c(C-C_\infty) \tag{1.14}$$

$$v\frac{\partial^2 g}{\partial z^2}-\frac{\partial g}{\partial t}-\Omega f+\Omega^2 l=0 \tag{1.15}$$

Using Equation(1.2) the corresponding initial and boundary conditions become:

$$f(z,0)=\Omega l,g(z,0)=0;\ \forall z>0, \tag{1.16}$$

$$f(o,t)=UH(t)\cos(\omega t)orf(0,t)=Usin(\omega t),\ \ g(o,t)=0;\ \forall t>0,$$

$$f(\infty,t)=\Omega l,g(\infty,0)=0;\ \forall t>0, \tag{1.17}$$

Now, we combine Equations (1.14) and (1.15), using $F=f+ig$ [29, 42] with the

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corresponding initial and boundary conditions (1.16) and (1.17) as follows:

$$v\frac{\partial^2 F}{\partial z^2}-\frac{\partial F}{\partial t}-i\Omega f+i\Omega^2 l=-g_x\beta_T(T-T_\infty)-g_x\beta_c(C-C_\infty) \quad (1.18)$$

$$F(z,0)=\Omega l;\forall z>0, \quad (1.19)$$

$$F(o,t)=UH(t)\cos(\omega t) or F(0,t)=Usin(\omega t), g(o,t)=0;\ \forall t>0,$$

$$F(\infty,t)=\Omega l;\forall z>0. \quad (1.20)$$

Introducing the following non-dimensional variables [11, 12, 23, 25, 31, 38, 42]:

$$F^*=\frac{F}{\Omega l}-1, z^*=\sqrt{\frac{\Omega}{v}}z, t^*=\Omega t, \omega^*=\frac{\omega}{\Omega}, T^*=\frac{T-T_\infty}{T_\omega-T_\infty} \quad (1.21)$$

$$c^*=\frac{C-C_\infty}{C_\omega-C_\infty}$$

The system of equations reduces to (dropping out the *notations)

$$\frac{\partial^2 F}{\partial z^2}-\frac{\partial F}{\partial t}-iF=-GrT-GmC$$

$$(1.22) F(z,o)=0,\forall z>0, F(o,t)=-1+U_0H(t)\cos(\omega t);or$$

$$F(o,t)=-1+Usin(\omega t), F(\infty,t)=0;\forall t>0, \quad (1.23)$$

$$\frac{\partial T}{\partial t}=\frac{1}{Pr}\frac{\partial^2 T}{\partial z^2}, \quad (1.24)$$

$$T(z,0)=0;\forall z>0, T(0,t)=1, T(\infty,t)=0;\forall z>0, \quad (1.25)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial z^2}, \quad (1.26)$$

$$C(z,0)=0;\forall z>0, C(0,t)=1, C(\infty,t)=0;\forall t>0, \quad (1.27)$$

Where $Gr=\frac{g_x\beta_T}{\Omega^2 l}(T_w-T_\infty), Gm=\frac{g_x\beta_C}{\Omega^2 l}(C_w-C_\infty), Pr=\frac{\mu c_p}{k}, Sc=\frac{v}{D}, U_0=\frac{U}{\Omega l}$

Here, Gr is the Grash of number, Gm. is modified Grash of number, Pr is Prandtl number, Sc is Schmidt number and U_0 is dimensionless parameter of amplitude of the plate oscillations.

Solution of the problem

In order to solve the system of Equations(1.22)- (1.27), we use Laplace transform method and obtain:

$$\frac{d^2\bar{F}}{dz^2}-(q+i)\bar{F}=-Gr\bar{T}-Gm\bar{C} \quad (1.28)$$

$$L\{F(0,t)\}=L[-1]+L[U_0H(t)cos\omega t]$$

$$\bar{F}(0,q)=-\frac{1}{q}+\frac{U_0 q}{q^2+\omega^2}$$

$$L[F(0,t)]=L[-1]+L(U_0 sin\omega t) Or$$

$$\bar{F}(0,q)=-\frac{1}{q}+U_0\frac{\omega}{q^2+\omega^2}, \bar{F}(\infty,q)=0 \quad (1.29)$$

$L\left[\frac{\partial T}{\partial t}\right] = q^T = \frac{1}{Pr}\frac{\partial^2 \bar{T}}{\partial z^2}$

$$\frac{d^2\bar{T}}{dz^2} - qPr\bar{T} = 0, \quad (1.30)$$

$L[T(0,t)] = \bar{T}(0,q) = L[1]$

$$\bar{T}(0,q) = \frac{1}{q}, \bar{T}(\infty,q) = 0 \quad (1.31)$$

$$\frac{d^2\bar{C}}{dz^2} - qSc\bar{C} = 0, \quad (1.32)$$

$$\bar{C}(0,q) = \frac{1}{q}, \bar{C}(\infty,q) = 0 \quad (1.33)$$

Now, Equations (1.28),(1.30) and (1.32) are solved using boundary condition (1.29), (1.31) and (1.33), and then, the inverse Laplace transforms of the resultant solutions are obtained as follows:

$$F_c(z,t) = F_1(z,t) - F_2(z,t) + F_3(z,t) + F_4(z,t) + F_5(z,t) - F_6(z,t) + F_7(z,t) - F_8(z,t) + F_9(z,t) \quad (1.34)$$

$$F_s(z,t) = F_1(z,t) - F_2(z,t) + F_3(z,t) + F_{10}(z,t) - F_{11}(z,t) - F_6(z,t) + F_7(z,t) - F_8(z,t) + F_9(z,t) \quad (1.35)$$

$$T(z,t) = erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}}\right) \quad (1.36)$$

$$C(z,t) = erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}\right) \quad (1.37)$$

$$F_1(z,t = \frac{b_2}{2}\exp(b_1 t)\left[\exp\left(-z\sqrt{b_1+i}\right) erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{(b_1+i)t}\right) + \exp\left(z\sqrt{b_1+i}\right) erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{(b_1+i)t}\right)\right]$$

$$F_2(z,t) = e_3\frac{1}{2}\left[\exp\left(-z\sqrt{i}\right) erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{it}\right) + \exp\left(z\sqrt{i}\right) erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{it}\right)\right]$$

$$F_3(z,t) = \frac{e_2}{2}\exp(e_1 t)\exp\left(-z\sqrt{e_1+i}\right) erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{(e_1+i)t}\right) + \exp\left(z\sqrt{e_1+i}\right) erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{(e_1+i)t}\right)\Big]$$

$$F_4(z,t) = \frac{b_2}{2}\mathrm{H(t)}\exp(i\omega t)\left[\exp\left(-z\sqrt{i\omega+i}\right) erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{i\omega t+it}\right) + \exp z\sqrt{i\omega+i}\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{(e_1+i)t}\right)\right]$$

$$F_5(z,t) = \frac{b_3}{2}\mathrm{H(t)}\exp(-i\omega t)\left[\exp\left(-z\sqrt{i-i\omega}\right) erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{it-i\omega t}\right) + \exp z\sqrt{i-i\omega}\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{it-i\omega t}\right)\right]$$

$$F_6(z,t) = \frac{b_2}{2}\exp(b_1 t)\left[\exp\left(-z\sqrt{Prb_1}\right) erfc\left(\frac{z}{2}\sqrt{\frac{Pr}{t}} - \sqrt{b_1 t}\right) + \exp z\sqrt{Prb_1}\, erfc\left(\frac{z}{2}\sqrt{\frac{Pr}{t}} + \sqrt{b_1 t}\right)\right]$$

$$F_7(z,t) = b_2 \mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{Pr}{t}}\right)$$

$$F_8(z,t) = \frac{e_2}{2}\exp(e_1 t)\left[\exp\left(-z\sqrt{e_1 Sc}\right) erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}\sqrt{e_1 t}\right) + \exp z\sqrt{e_1 Sc}\, erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}} + \sqrt{e_1 t}\right)\right]$$

$$F_9(z,t) = e_2 \mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}\right)$$

$$F_{10}(z,t) = \frac{b_7}{2}\exp(i\omega t)\left[\exp\left(-z\sqrt{i\omega+i}\right) erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{i\omega t+it}\right) + \exp z\sqrt{i\omega+i}\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{i\omega t+it}\right)\right]$$

$$F_{11}(z,t) = \frac{b_7}{2}\exp(-i\omega t)\left[\exp\left(-z\sqrt{i-i\omega}\right) erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{it-i\omega t}\right) + \exp(z\sqrt{i-i\omega} erfc\left(+\sqrt{it-i\omega t}\right)\right]$$

Where $a_1 = Pr-1, b_1 = \frac{i}{a_1},\ a_2 = Sc-1, e_1 = \frac{i}{a_2}, b_2 = \frac{Gr}{a_1 b_1}, b_3 = \frac{U_0}{2},$ $e_2 = \frac{Gm}{a_1 e_1}, e_3 = b_2 + e_2 + 1\ and\ b_7 = \frac{U_0}{2i}.$

It is important to note that solutions (1.34) and (1.35) are not valid for Pr=1 or Sc=1 as well as for Pr=1 and Sc=1. Therefore, we calculate separately solution of velocity for these cases in the following:

When$Pr = 1\ and\ Sc \neq 1$:

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$$F_c(z,t) = F_3(z,t) - F_{12}(z,t) + F_4(z,t) + F_5(z,t) + F_{13}(z,t) - F_8(z,t) + F_9(z,t) \quad (1.38)$$

$$F_s(z,t) = F_3(z,t) - F_{12}(z,t) + F_{10}(z,t) - F_{11}(z,t) + F_{13}(z,t) - F_8(z,t) + F_9(z,t) \quad (1.39)$$

Where

$$F_{12}(z,t) = \frac{e_4}{2}\left[\exp(-z\sqrt{i})\, erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{it}\right) + \exp(z\sqrt{i})\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{it}\right)\right]$$

$$F_{13}(z,t) = b_5 \text{erfc}\left(\frac{z}{2\sqrt{t}}\right)$$

Where $b_5 = \frac{Gr}{i}$ $and\ e_4 = b_5 + e_2 + 1$

When Sc=1 and $Pr \neq 1$

$$F_c(z,t) = F_1(z,t) - F_{14}(z,t) + F_4(z,t) + F_5(z,t) - F_6(z,t) + F_7(z,t) + F_{15}(z,t) \quad (1.40)$$

$$F_s(z,t) = F_1(z,t) - F_{14}(z,t) + F_{10}(z,t) - F_{11}(z,t) - F_6(z,t) + F_7(z,t) + F_{15}(z,t) \quad (1.41)$$

$$F_{14}(z,t) = \frac{e_6}{2}\left[\exp(-z\sqrt{i})\, erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{it}\right) + \exp(z\sqrt{i})\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{it}\right)\right],$$

$$F_{15}(z,t) = e_5 \text{erfc}\left(\frac{z}{2\sqrt{t}}\right)$$

Where $e_5 = \frac{Gm}{i}$ $and\ e_6 = b_2 + e_5 + 1$

When Pr=1 and Sc=1:

$$F_c(z,t) = F_4(z,t) + F_5(z,t) - F_{16}(z,t) \quad (1.42)$$

$$F_s(z,t) = F_{10}(z,t) - F_{11}(z,t) - F_{16}(z,t) \quad (1.43)$$

$$F_{14}(z,t) = \frac{b_8}{2}\left[\exp(-z\sqrt{i})\, erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{it}\right) + \exp(z\sqrt{i})\, erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{it}\right)\right], \quad (1.44)$$

Where, $b_8 = b_5 + e_5 + 1$.

Skin friction, Nusselt number and Sherwood number

The expressions of the dimensional skin friction are given by [11, 13, 42]:

$$\tau = -\left[\mu \frac{\partial F}{\partial z}\right]_{z=0} \quad (1.45)$$

Which in non-dimensional form reduces to:

$$\tau^* = -\left[\mu \frac{\partial F^*}{\partial z^*}\right]_{z^*=0} \quad (1.46)$$

Where $\tau^* = \frac{\sqrt{v}}{\mu l \Omega^{\frac{3}{2}}}\tau$ finally, Equation.(1.46), in view of equations (1.34) and (1.35), gives (dropping out the * notation):

$$\tau_c(z,t) = \tau_1(z,t) - \tau_2(z,t) + \tau_3(z,t) + \tau_4(z,t) + \tau_5(z,t) - \tau_6(z,t) + \tau_7(z,t) - \tau_8(z,t) + \tau_9(z,t) \quad (1.47)$$

$$\tau_s(z,t) = \tau_1(z,t) - \tau_2(z,t) + \tau_3(z,t) + \tau_{10}(z,t) - \tau_{11}(z,t) - \tau_6(z,t) + \tau_7(z,t) -$$

$$\tau_8(z,t)+\tau_9(z,t) \tag{1.48}$$

$$\tau_1(z,t)=\frac{b_2}{2}\frac{\exp(b_1t)}{2}[\sqrt{b_1+i}erfc(-\sqrt{b_1t+it})-\sqrt{b_1+i}\,erfc\,\frac{2}{\sqrt{\pi t}}\exp(-(b_1t+it))]$$

$$\tau_2(z,t)=-e_3\frac{1}{2}\left[\sqrt{i}erfc(-\sqrt{it})-\sqrt{i}erfc\left(\sqrt{it}\right)+\frac{2}{\sqrt{\pi t}}\exp(-it)\right]$$

$$\tau_3(z,t)=-e_2\frac{\exp(e_1t)}{2}\left[\sqrt{e_1+i}erfc\left(\sqrt{e_1t+it}\right)\right.$$

$$\left.-\sqrt{e_1+i}erfc\left(\sqrt{e_1t+it}\right)+\frac{2}{\sqrt{\pi t}}\exp(-(e_1t+it)\right]$$

$$\tau_4(z,t)=-b_3H(t)\frac{\exp(i\omega t)}{2}\left[\sqrt{i+\omega}ierfc\left(-\sqrt{it+i\omega t}\right)\right.$$

$$\left.-\sqrt{i+\omega i}\,erfc\left(\sqrt{it+i\omega t}\right)+\frac{2}{\sqrt{\pi t}}\exp(-(it+i\omega t)\right]$$

$$\tau_5(z,t)=-\,b_3\,H(t)\frac{\exp(-i\omega t)}{2}\left[\sqrt{i-\omega}ierfc\left(-\sqrt{it-i\omega t}\right)\right.$$

$$\left.-\sqrt{i-\omega i}\,erfc\left(\sqrt{it-i\omega t}\right)+\frac{2}{\sqrt{\pi t}}\exp(-(it-i\omega t)\right]$$

$$\tau_6(z,t)=-b_2\frac{\exp(b_1t)}{2}\left[\sqrt{Prb_1}erfc\left(-\sqrt{b_1t}\right)\right.$$

$$\left.-\sqrt{Prb_1}erfc\left(\sqrt{b_1t}\right)+2\sqrt{\frac{Pr}{\pi t}}\exp(-(it-i\omega t))\right]\tau_7(z,t)=b_2\left(\sqrt{\frac{Pr}{\pi t}}\right)$$

$$\tau_8(z,t)=-e_2\frac{\exp(e_1t)}{2}\left[\sqrt{e_1Sc}erfc\left(-\sqrt{e_1t}\right)-\sqrt{e_1Sc}erfc\left(\sqrt{e_1t}\right)-2\sqrt{\frac{Sc}{\pi t}}\exp(-e_1t)\right]$$

$$\tau_9(z,t)=e_2\sqrt{\frac{Sc}{\pi t}}$$

$$\tau_{10}(z,t)=-b_7\frac{\exp(i\omega t)}{2}[\sqrt{i+i\omega}\,erfc\left(-\sqrt{it+i\omega t}\right)$$

$$+\sqrt{i+\omega}ierfc(\sqrt{it+i\omega t})+\frac{2}{\pi t}\exp\left(-(it+i\omega t)\right)]$$

$$\tau_{11}(t)$$

$$=-b_7\frac{\exp(i\omega t)}{2}[\sqrt{i-i\omega}\,erfc\left(-\sqrt{it-i\omega t}\right)\sqrt{i-\omega}ierfc(\sqrt{it-i\omega t})+\frac{2}{\pi t}\exp\left(-(it\right.$$

$$\left.-i\omega t)\right)]$$

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The rate of heat transfer (Nusselt number) and rate of mass transfer (Sherwood number) are given as [11, 13, and 42]:

$$Nu = \left[\frac{\partial T}{\partial z}\right]_{z=0} \tag{1.49}$$

$$Nu = \frac{\sqrt{Pr}}{\sqrt{\pi t}} \tag{1.50}$$

$$Sh = \left[\frac{\partial C}{\partial z}\right]_{z=0} \tag{1.51}$$

$$Sh = \frac{\sqrt{Sc}}{\sqrt{\pi t}} \tag{1.52}$$

MATLAB Coding

```
%Velocity profile
clc;
ncx = complex(0,1);
t= 2.50;
Pr = 0.710;
Sc = 0.60;
omga = 0.0;
U0 = 3.0;
Gm = 5.0;
Gr = 5.0;
Za = 0.0:0.01:5.0;
z= Za';
a1 = Pr - 1.0;
b1 = ncx/a1;
a2 = Sc - 1.0;
e1 = ncx/a2;
b2 = Gr/(a1*b1);
b3 = U0/2.0;
e2 = Gm/(a2*e1);
e3 = b2 + e2 + 1.0;
b7 = U0/(2.0*ncx);
F1 = ((b2*exp(b1*t))/2.0)*( exp(-z.*sqrt(b1 + ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) -sqrt((b1 +
ncx)*t) ) )+exp(z.*sqrt(b1 + ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) +sqrt((b1 + ncx)*t))));
F2 = (e3/2.0)*( exp(-z.*sqrt(ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) -sqrt( ncx*t) ))+exp(z.*sqrt(
```

```
ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) +sqrt( ncx*t) )) );
F3 = ((e2*exp(e1*t))/2.0)*( exp(-z.*sqrt(e1 + ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) -sqrt((e1 + ncx)*t) ))+exp(z.*sqrt(e1 + ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) +sqrt((e1 + ncx)*t) )) );
F4 =((b3*Heaviside(t)*exp(ncx*omga*t))/2.0)*( exp(-z.*sqrt(omga*ncx +ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) -sqrt((omga*ncx + ncx)*t) ))+exp(z.*sqrt(omga*ncx + ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) +sqrt((omga*ncx + ncx)*t) )) );
F5 =((b3*Heaviside(t)*exp(-ncx*omga*t))/2.0)*( exp(-z.*sqrt(-omga*ncx +ncx)).*( 1.0 - erfz((z./(2.0*sqrt(t))) -sqrt((-omga*ncx + ncx)*t) ))+exp(z.*sqrt(-omga*ncx + ncx)).*(1.0 - erfz((z./(2.0*sqrt(t))) +sqrt((-omga*ncx + ncx)*t) )) );
F6=((b2*exp(b1*t))/2.0)*(exp(-z.*sqrt(Pr*b1)).*(1.0 - erfz((z./2.0).*sqrt(Pr/t) -sqrt(b1*t) ))+exp(z.*sqrt(Pr*b1)).*(1.0 - erfz((z./2.0).*sqrt(Pr/t) +sqrt(b1*t) )) );
F7 =b2* erfc((z./2.0).*sqrt(Pr/t));
F8=((e2*exp(e1*t))/2.0)*( exp(-z.*sqrt(Sc*e1)).*(1.0 - erfz((z./2.0).*sqrt(Sc/t) -sqrt(e1*t) ))+exp(z.*sqrt(Sc*e1)).*(1.0 - erfz((z./2.0).*sqrt(Sc/t) +sqrt(e1*t) )) );
F9 = e2* erfc((z./2.0).*(sqrt(Sc/t)) );
Fc = F1 - F2 + F3 + F4 + F5 - F6 + F7 - F8 + F9;
disp(' zFc')
disp([zFc ])
holdon
plot(z, real(Fc), z, abs(image(Fc)))
holdoff
```

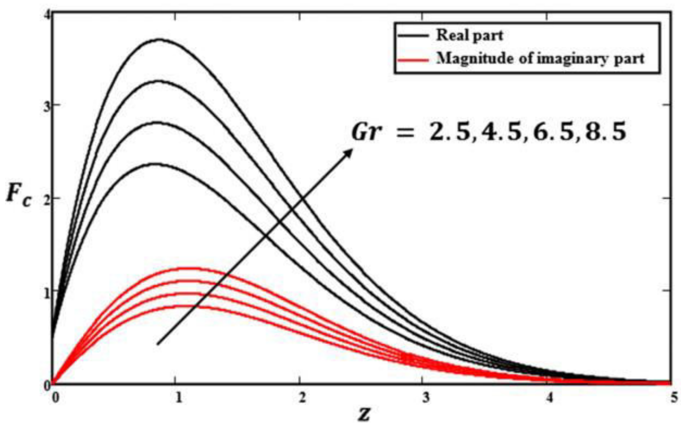

Fig 1 Velocity profiles for different values of G_r with $t = 1.0, P_r = 0.71,$

$$\omega = \frac{\pi}{3}, U_0 = 3.0, G_m = 5.0 \text{ and } S_c = 0.6$$

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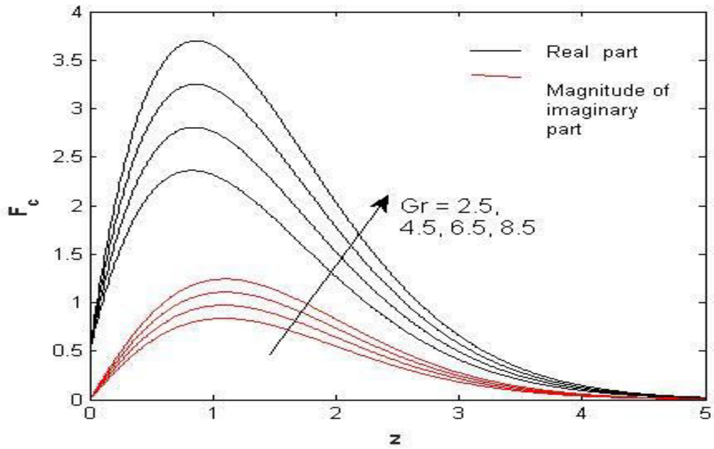

Fig 2:Velocity profiles values of P_r with $t = 1.0, G_r = 5.0$

$$\omega = \frac{\pi}{3}, U_0 = 3.0, G_m = 5.0 \; and \; S_c = 0.6$$

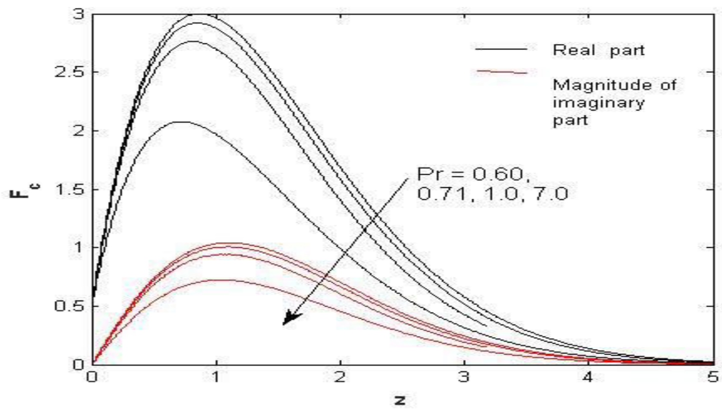

Fig 3 Velocity profiles values of G_m with $t = 1.0, G_r = 5.0, P_r = 0.71,$

$$\omega = \frac{\pi}{3}, U_0 = 3.0, and \; S_c = 0.6$$

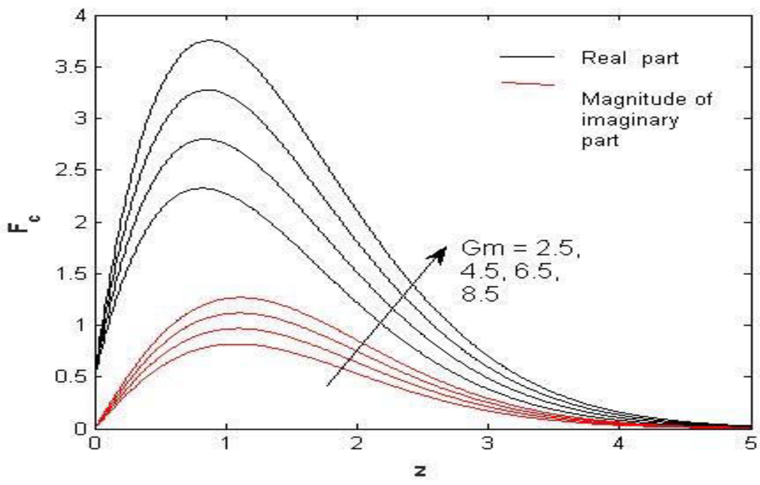

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Fig 4:Velocity profiles values of S_c with $t = 1.0, G_r = 5.0$

$$\omega = \frac{\pi}{3}, U_0 = 3.0, G_m = 5.0 \; and \; P_r = 0.6$$

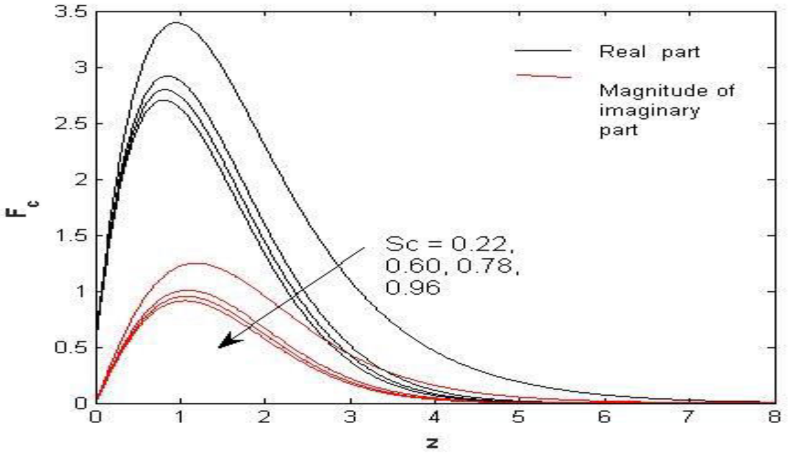

Fig 5 :Velocity profiles values of ωt with $t = 1.0, G_r = 5.0, S_c = 0.6,$

$$U_0 = 3.0, G_m = 5.0 \; and \; P_r = 0.71$$

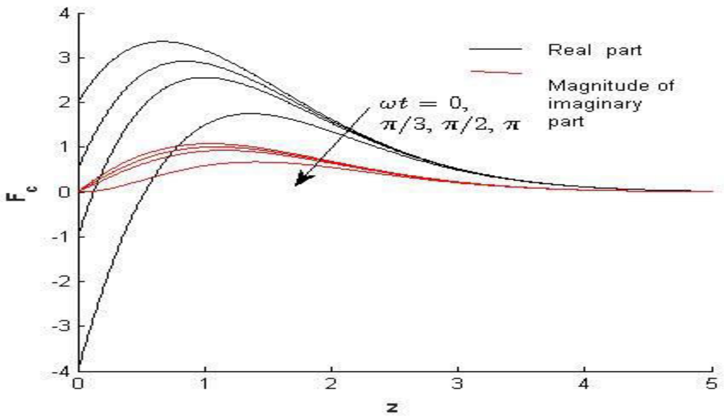

Fig 6:Velocity profiles for different values of $\omega t$with

$$t = 1.0, Gr = 5.0, \quad Sc = 0.6, U_0 = 3.0, Gm = 5.0 \; and \; Pr = 0.71$$

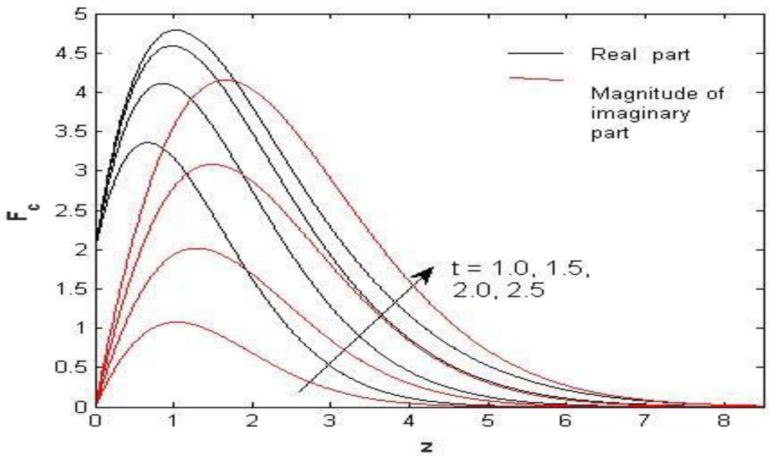

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Fig 7:Velocity profiles for different values of $t$with

$U_0 = 3.0, Gr = 5.0, \quad Sc = 0.6, \omega = 0, Gm = 5.0 and\ Pr = 0.71$

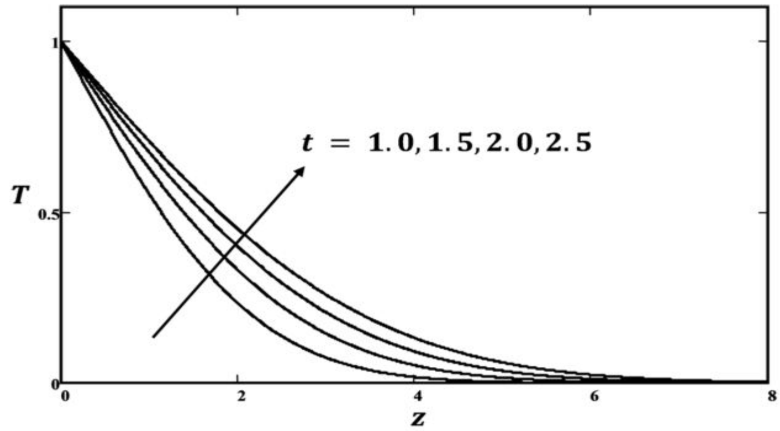

Fig 8 Temperature profiles for the different values oft with $P_r = 0.71$

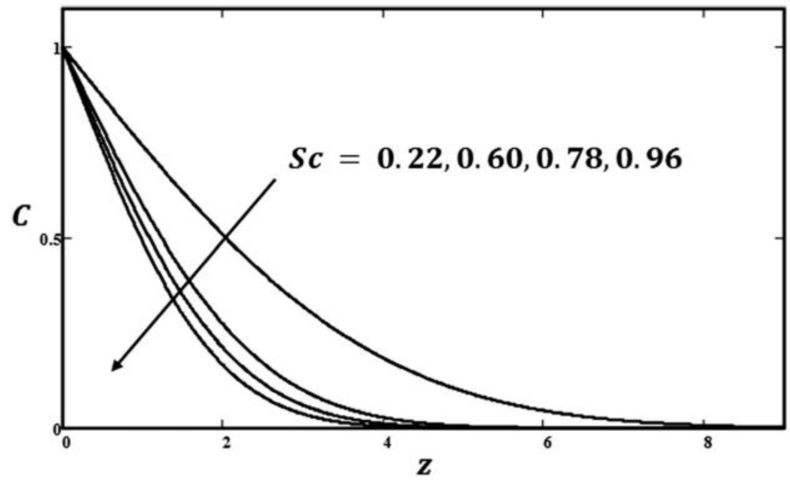

Fig9:Concentrationprofilesfor the different values of Sc with $t = 1.0$

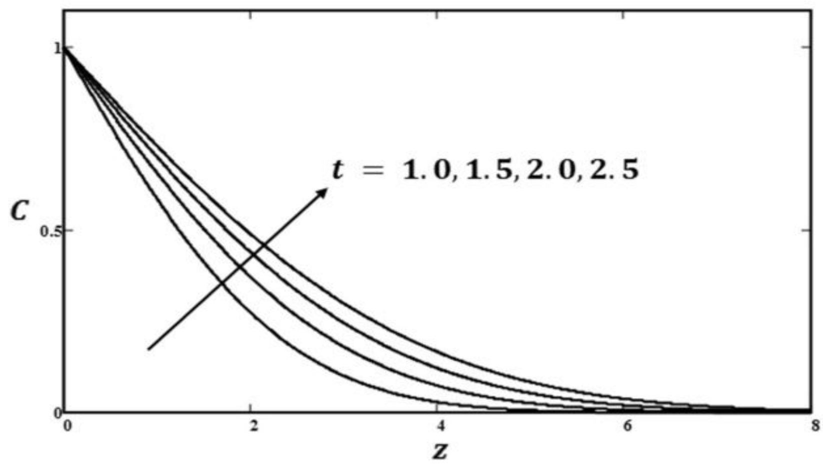

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Fig 10 Concentrationprofilesfor the different values of $t$with $Sc = 0.60$

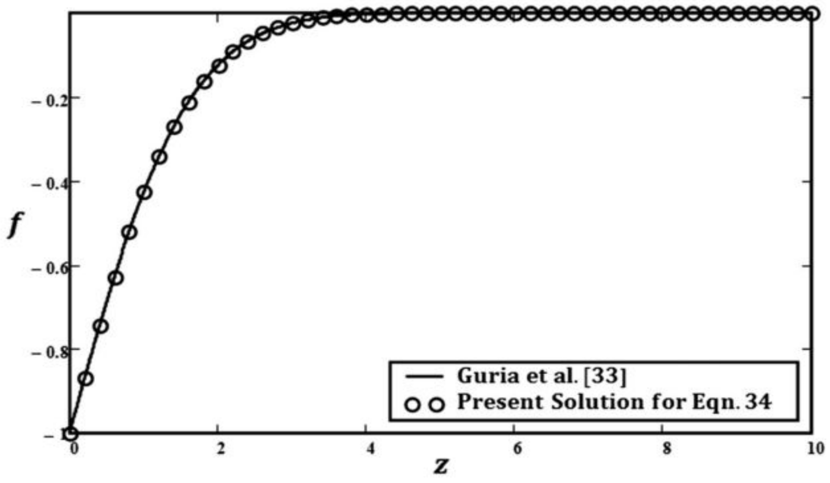

Fig 11:Comparison of primary velocity in present solution with Guria et al[33]

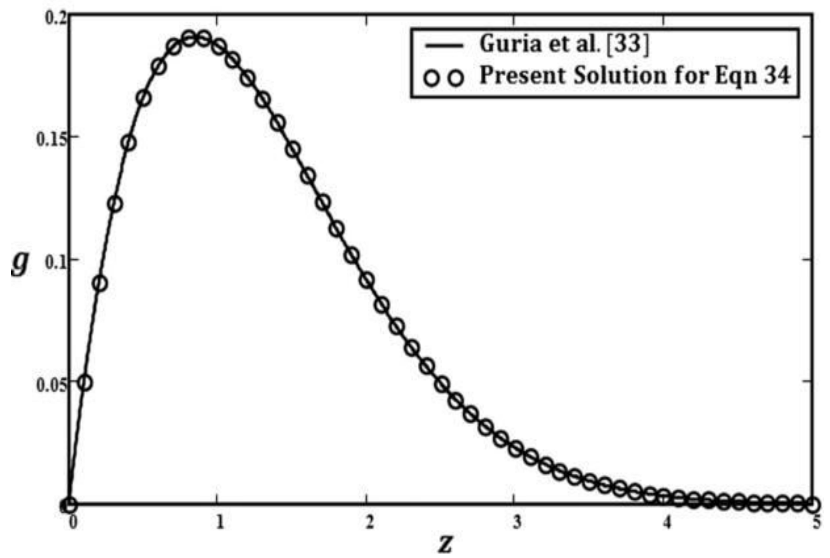

Fig 12:Comparison of secondaryvelocity in present solution withGuria et al[33]

3. Results and Discussion

Exact solution of heat and mass transfer for incompressible viscous fluid with non-coaxial rotation through an oscillating disk is obtained. In order to get the physics of the regime, the effects of various parameter such a Grashof number, Prandtl number, modified Grashof number, Schmidt number.

4. Summary and Conclusion

An exact solution for unsteady mixed convection flow of viscous fluid due to non-coaxial rotation over an oscillating vertical disk with isothermal temperature and constant mass diffusion is obtained using the Laplace transform method. Effects of various embedded parameters on velocity, temperature and concentration are studied graphically in various plots. Results of Skin friction, Nusselt number and Sherwood number are computed in

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different tables. The disk and fluid are rotating with uniform angular velocity which is equal to 1 in the present computations. The following main results are concluded from this study:

1. Both primary and secondary velocities increase with increasing Gr, Gm. and t.
2. Both primary and secondary velocities decrease with increasing Pr, Sc and ωt.
3. Temperature increases with increasing t and decreases when Pr is increased.
4. Concentration increases with increasing t and decreases when Sc is increased.
5. Skin friction increases with increasing values of Pr, Sc and ωt whereas it decreases with increasing values of Gr, Gm. and t
6. Nusselt number increases for increasing Pr and decreases for increasing t.
7. Sherwood number increases for increasing Sc and decreases for increasing t.

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